was held with a tight press fit in an outer jacket and, to investigate the effect of the latter, tests were made both with the outer jacket of the comparison assembly of the same material (tungsten) as the inner block, and also with a jacket of high tensile steel. These two arrangements showed no appreciable difference as regards the distortion factor.

All diametral measurements required on the pistons and cylinders were carried out in the Engineering Metrology Section of the Standards Division of the National Physical Laboratory by direct comparison with high quality slip gauges, the sizes of which are known to about $\pm 10^{-6}$ in ($\pm 0.025\ \mu$m) (NPL Ann. Rep. 1919; TAYLERSON 1955).

The main part of the load on the piston was applied in the familiar manner by annular masses stacked on a cylindrical carrier of the overhang type supported on the upper end of the piston by a steel ball. In order to minimise friction the assemblies were always operated with the piston and load system in free rotation. The speed of rotation is not in general critical for assemblies of the types used in the present measurements but for definiteness a speed in the range 30 – 40 rev/min was normally adopted. Piston-cylinder assemblies occasionally exhibit anomalous effects due to small helical errors on the piston surface – often referred to as "corkscrewing" – which have the effect of adding a spurious component – positive or negative according to the direction of rotation – to the load. These effects are easily identified and in order to eliminate them measurements were always made using both directions of rotation, and the mean value adopted. Any assembly showing a considerable degree of asymmetry of this kind would have been rejected as unsuitable for measurements of the accuracy and reproducibility necessary for the present work.

In carrying out the balancing experiments the fall of the pistons was observed either by the use of optical magnification, or electronically using a capacitance method.

The normal practice in taking observations over any given range of pressure was first to take a series in rising order of pressure and to follow this as soon as possible by a repeat in descending order. In general these series showed no systematic divergence and hysteresis effects were negligible. There was, however, one exception to this rule, applying to comparisons involving the tungsten base material at pressures above about 3000 bars. In this case the rising series of points over the upper part of the pressure range showed a tendency to curve away from the initial straight line in the sense of an abnormally large increase in area on the part of the tungsten assembly. This abnormal component of the deformation recovered only very slowly on removal of the pressure, and it was found that if, after exposure to the maximum pressure, a relatively rapid series of readings was taken in descending order, these approximated well to a straight line which, moreover, was sensibly parallel to the initial portion where hysteresis was not appreciable. As already pointed out, the elastic constant measurements on the tungsten base alloy showed very similar characteristics, with anelastic effects over the higher ranges of stress but providing reasonably consistent values of the elastic modulus from the series of readings taken with diminishing stress. It was considered justifiable, therefore, to regard the descending series as being fairly representative of the elastic behaviour of these assemblies, in so far as this enters into the similarity procedure. On this basis measurements with the steel and tungsten assemblies were extended up to the region of 6000 bars. The practicability of using some more recently developed alloys of high modulus is being considered for possible further extensions of the method.

4. Results of the Similarity Method

a) Measurements involving two materials for the range up to 3000 bars

Some account of the earlier measurements in this series has been given in two former papers (DADSON 1955, 1958) but for completeness the main features are summarised below.

Fig. 3 illustrates the results obtained with a series of piston-cylinder assemblies of type a) – Fig. 2 – covering three different ranges of pressure. The changes in effective area are shown as parts in 10^5 of the area at zero pressure, and in two cases results are given for different transmitting fluids.

As was mentioned earlier the distortion factors for assemblies of this type may be very closely represented as linear functions of the applied pressure, the dispersion of the experimental points rarely amounting to more than $\pm$ 1 part in 10^5.

It will also be apparent that for a given fluid the distortion coefficients for assemblies having different cylinder bores are very similar, the coefficient λ_S being normally in the region 4×10^{-7}/bar. The normal manufacturing tolerances on this type of assembly seem to involve little variation in the distortion coefficient, the values for a substantial group for the same transmitting fluid having been found to vary by only a few percent.

A point of interest arises in connection with the use of different fluids, when, as illustrated in Fig. 3,

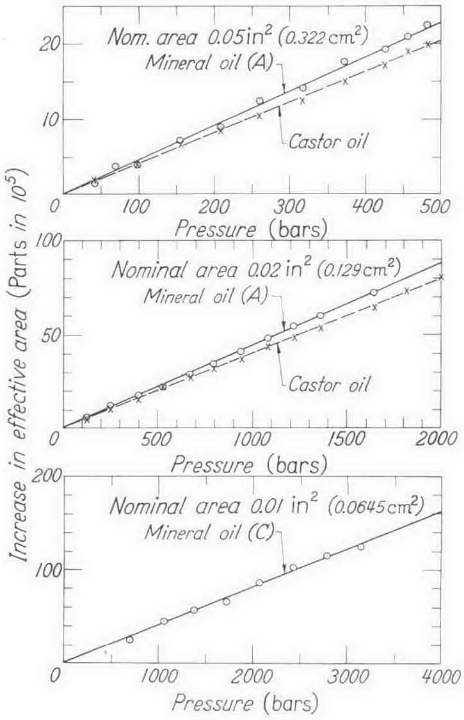

Fig. 3.
Distortion factors of a group of steel piston-cylinder assemblies of type a

some variation of the distortion coefficient may occur. It would seem that these effects must be connected with differences in the functional form of the dependence of the coefficient of viscosity upon pressure and its resulting influence on the pressure distribution in the interspace between piston and cylinder. In the discussion of the formal theory of the pressure balance earlier in this paper the effect was examined of assuming that the components of the radial displacements of the surfaces of the piston and cylinder at a given position due to the fluid pressure in the interspace could be taken as proportional to the pressure at the same position. Reasons were adduced that this assumption was unlikely to be much in error in the case of the piston, but was less secure in the case of the cylinder. It is an immediate consequence of this assumption – see equation (2.6) – that the distortion factor is independent of the actual pressure distribution in the interspace, and should therefore be independent of the transmitting fluid. The experimental results thus provide evidence that the assumption in question is not entirely correct, at least for the assemblies of type a), and it will be the cylinder, the lateral dimensions of which are not small compared with the length of the working section of the bore, where the principal limitation will arise. If the cylinders were appreciably longer compared with their wall thickness, and the region of attachment were located further away from the working portion, the dependence on the nature of the fluid might well be reduced. Although the changes so far observed are not very large, they are sufficient to require that any standard calibration of a piston-cylinder assembly intended for work of high accuracy must be associated with the particular fluid used. This is an aspect of the pressure balance on which more data would be useful.

b) Results of measurements involving three materials with discussion of errors

The three-material procedure has been carried out for two pressure ranges — 500 and 1200 bars — employing assemblies of type a) — Fig. 2 — of nominal areas 0.05 and 0.02 in^2 respectively, using castor oil as the pressure transmitting fluid. The results of these measurements are summarised in Tab. 2 in which are shown the values of the distortion coefficients for the steel assemblies determined both by the direct and indirect methods. Over the pressure range in question the dependence of disrortion on pressure was closely linear, with no appreriable hysteresis effects. The actual coefficients given are best fits by least squares to some four to six sets of data. It is worthy of note that it has been verified by direct balancing that the distortion coefficients of the two steel assemblies concerned are actually equal to within 1%. The total dispersion of the results is in the region $\pm$ 4%, but it will be seen that there is evidence that the direct comparisons involving bronze (λ'_S) are subject to more scatter than the remainder. This result is not surprising since, from the point of view of the influence of possible uncertainties in the elastic constants, this comparison is in every way at a disadvantage relative to the other two. Since the factor k has here its smallest value ($= 1.44$), and the comparison is with an assembly having a *larger* distortion, the operative factor in equation (3.6), viz $(k - 1)$, is particularly sensitive to an error in k. The fact that the Poisson's ratios are somewbat different is also not an advantage, although the correction factor already discussed should take account of this. Making use of equations (3.6) to (3.8) and introducing the actual numerical values of k_{BS} ..., it is easily shown that an error of x% in the relevant ratio of elastic moduli (k) leads to percentage errors in the three values, λ'_S, λ''_S and λ'''_S of the distortion factor, of about 3.3x, 1.4x and 0.9x respectively. In this respect therefore, the direct comparison using S and T and the indirect comparison, would be expected to show an appreciable advantage over the direct comparison using S and B. Considering now the errors associated with the correction terms θ_S ... of equations (3.6) to (3.8), introduced to allow for differences of Poisson's ratio, some advantage may lie with the direct comparison using S and T in which the two Poisson's ratios are nearly equal, the correction term in this case amounting to only about 2% of the total distortion factor.

The data of Tab. 2 are therefore seen to be consistent with the assumptions that the main errors involved are associated with the values adopted for the elastic moduli, and that the ratios of these are known to the order of $\pm$ 1 or 2%, the corresponding distortion coefficients being contained within a dispersion of about $\pm$ 4%. If, however, the two most favourable comparisons (λ''_S and λ'''_S) are selected, and the mean taken, the final result is unlikely to be in error by more than about 2%. In the practical application of the results this procedure has been adopted.

Table 2. *Results of three-material experiments*

Nominal effective area	Pressure range (bars)	Distortion coefficient of steel assembly for castor oil (bar^{-1})		
		Direct comparison with bronze (λ'_S)	Direct comparison with tungsten (λ''_S)	Indirect comparison (λ'''_S)
0.05 in^2 (0.322 cm^2 approx.)	500	$4.2_1 \times 10^{-7}$	$4.0_0 \times 10^{-7}$	$4.0_9 \times 10^{-7}$
0.02 in^2 (0.129 cm^2 approx.)	1200	$3.9_6 \times 10^{-7}$	$4.1_0 \times 10^{7}$	$4.0_5 \times 10^{-7}$
Mean results for above cases		$4.0_8 \times 10^{-7}$	$4.0_5 \times 10^{-7}$	$4.0_7 \times 10^{-7}$

c) Extension to pressures of 6000 *bars*

The extension of the similarity method from 3000 to the region of 6000 bars has been carried out entirely with assemblies of type b), of nominal area 0.005 in^2, those of type a) being normally restricted to use below 3000 bars. The experimental value of the distortion coefficient is $3.0_2 \times 10^{-7}$/bar, and is thus appreciably smaller than the figure for assemblies of type a) averaging at about $4.0_6 \times 10^{-7}$/bar.

The form of the type b) assemblies approximates more closely to the "ideal" piston-cylinder combination. In considering the formal theory in Section 2 it was noted that a very simple approximation to the distortion factor could be derived on the assumption that the radial displacements of the piston and cylinder surfaces at any position due to the fluid pressure in the interspace are proportional to the pressure at that position, and the limitations of this assumption were discussed. Inserting the appropriate numerical values in equation (2.6) the distortion coefficient so deduced, assuming a ratio of external to internal cylinder diameter of 10:1, is about 2.9×10^{-7}/bar. The close approach of this figure to the experimental value for the type b) assemblies certainly suggests that the assumptions involved in the "naive" theory are not greatly in error in this case. There are, however, some features of the actual cylinder, notably the